Belief Logic

u:A	=	You believe that A.
	=	You accept A.
u:A	=	Believe that A.
	=	Accept A.

1. The result of writing a small letter and then ":" and then a wff is a *descriptive* wff.
2. The result of writing an underlined small letter and then ":" and then a wff is an *imperative* wff.

You believe that A is true	=	u:A
You don't believe that A is true	=	~u:A
You believe that A is false	=	u:~A
You don't believe A and you don't believe not-A	=	(~u:A • ~u:~A)
You believe that you ought to do A	=	u:OA$\underline{u}$
Everyone believes that they ought to do A	=	(x)x:OA$\underline{x}$
You believe that if A then not-B	=	u:(A ⊃ ~B)
If you believe A, then you don't believe B	=	(u:A ⊃ ~u:B)

Believe that A is true = $\underline{u}$:A

Don't believe that A is true = ~$\underline{u}$:A

Believe that A is false = $\underline{u}$:~A

Don't believe A and don't believe not-A = (~$\underline{u}$:A • ~$\underline{u}$:~A)

Believe that you ought to do A = $\underline{u}$:OA$\underline{u}$

Let everyone believe that they ought to do A = (x)$\underline{x}$:OA$\underline{x}$

If you in fact believe A, then don't believe B = (u:A ⊃ ~$\underline{u}$:B)

Don't combine believing A with believing B = ~($\underline{u}$:A • $\underline{u}$:B)

Three Approaches to Belief Logic

1. Belief logic studies what belief formulas validly follow from what other belief formulas.
2. Belief logic studies how people would believe if they were *completely consistent*.
3. Belief logic generates *consistency imperatives*, like:
   - "Don't combine believing A with believing not-A"
     "~(u:A • u:~A)"
   - "Don't combine believing A-and-B with not believing A"
     "~(u:(A • B) • ~u: A)"

Belief worlds are represented by strings of one or more instances of a small letter – for example, "u," "uu," "uuu," and so on.

A *belief policy* is a set of imperatives about what you are or are not to believe, e.g., u:P, ~u:L, ~u:~L ("Believe that Michigan will play; be neutral about whether Michigan will lose").

Belief logic forbids belief policies that tell you to believe inconsistently (where what you're told to believe is inconsistent or else logically entails something that you're told not to believe). These forbidden belief policies are ones where these two rules lead to some belief world being inconsistent (apply B- before B+):

B- If you're told to *refrain* from believing A, then put not-A in a *new* belief world of yours.

B+ If you're told to believe A, then put A in all of your belief worlds.

Belief Inference Rules

B- | ~u:A → u ∴ ~A, use a *new* string of u's | * | First drop negative imperative belief operators; use a new belief world each time.

B+ | u:A → u ∴ A, use any string of u's | | Then drop positive imperative belief operators; use old belief worlds if you have them (otherwise use a new world "u").

"Don't combine believing A with believing not-A."

[∴ ~(u:A • u:~A) **Valid**

* 1 ┌ asm: (u:A • u:~A)
2 │ ∴u:A {from 1}
3 │ ∴u:~A {from 1}
4 │ **u ∴ A {from 2} ⇦ B+**
5 └ **u ∴ ~A {from 3} ⇦ B+**
6 ∴ ~(u:A • u:~A) {from 1; 4 contradicts 5}

Apply B- before B+

B- If you're told to *refrain* from believing A, then put not-A in a *new* belief world of yours.

B+ If you're told to believe A, then put A in all of your belief worlds.

"Don't combine believing A-and-B with not believing A."

[∴ ~(u:(A • B) • ~u:A) **Valid**

* 1 ┌ asm: (u:(A • B) • ~u:A)
2 │ ∴u:(A • B) {from 1}
* 3 │ ∴~u:A {from 1}
4 │ **u ∴ ~A {from 3} ⇦ B-**
5 │ u ∴ (A • B) {from 2} ⇦ **B+**
6 └ **u ∴ A {from 5}**
7 ∴ ~(u:(A • B) • ~u:A) {from 1; 4 contradicts 6}

Apply B- before B+

B- If you're told to *refrain* from believing A, then put not-A in a *new* belief world of yours.

B+ If you're told to believe A, then put A in all of your belief worlds.

1 □(A ⊃ B) Valid

[∴ ~(u:A • ~u:B)

* 2 ┌ asm: (u:A • ~u:B)

3 │ ∴ u:A {from 2}

* 4 │ ∴ ~u:B {from 2}

5 │ **u ∴ ~B {from 4}** ⇦ **B-**

6 │ u ∴ A {from 3} ⇦ **B+**

* 7 │ u ∴ (A ⊃ B) {from 1}

8 └ **u ∴ B {from 6 and 7}**

9 ∴ ~(u:A • ~u:B) {from 2; 5 contradicts 8}

1. Reverse squiggles (quantificational/modal/deontic).
2. Drop weak operators, using new things: ~u: R (∃x) ◇
3. Lastly, drop strong operators, using old things (if you have them): u: O (x) □

	u:A	=	You accept (endorse, assent to, say in your heart) "A is true."
		=	You believe that A.
	u:<u>A</u>	=	You accept (endorse, assent to, say in your heart) "Let act A be done."
		=	You will that act A be done.
If A is present:	u:A<u>u</u>	=	You accept the imperative for you to do A now.
		=	You act (in order) to do A.
If A is future:	u:A<u>u</u>	=	You accept the imperative for you to do A in the future.
		=	You're resolved to do A.
If $u \neq x$:	u:A<u>x</u>	=	You accept the imperative for X to do A.
		=	You desire (or want) that X do A.

u:A$\underline{u}$	=	You act (in order) to do A.
	=	You say in your heart, "Do A" (addressed to yourself).

Au	=	You do A.

u:(∃x)(Kx · R$\underline{x}$)	=	You desire that some who kill *repent*.
	=	You say in your heart "Would that some who kill *repent*."
u:(∃x)(K$\underline{x}$ · Rx)	=	You desire that some *kill* who repent.
	=	You say in your heart "Would that some *kill* who repent."
u:(∃x)(K$\underline{x}$ · R$\underline{x}$)	=	You desire that some both *kill* and *repent*.
	=	You say in your heart "Would that some *kill* and *repent*."

	u:A	=	Accept (endorse, assent to, say in your heart) "Let act A be done."
		=	Will that act A be done.
If A is present:	u:Au	=	Accept the imperative for you to do A now.
		=	Act (in order) to do A.
If A is future:	u:Au	=	Accept the imperative for you to do A in the future.
		=	Be resolved to do A.
If u≠x:	u:Ax	=	Accept the imperative for X to do A.
		=	Desire (or want) that X do A.

Use underlining *before* ":" to *tell* someone what to believe or will.

Use underlining *after* ":" if the sentence is about *willing*.

Indicatives		
u:A	=	You believe A.
u:<u>A</u>	=	You will A.

Imperatives		
<u>u</u>:A	=	Believe A.
<u>u</u>:<u>A</u>	=	Will A.

Don’t combine *believing* that it’s wrong for you to do A with *acting* to do A.

[∴ ~(u:O~Au • u:Au) Valid

* 1 ┌ asm: (u:O~Au • u:Au)
2 │ ∴ u:O~Au {from 1}
3 │ ∴ u:Au {from 1}
4 │ u ∴ O~Au {from 2} ⇦ **B+**
5 │ **u ∴ Au {from 3}** ⇦ **B+**
6 └ **u ∴ ~Au {from 4}**
7 ∴ ~(u:O~Au • u:Au) {from 1; 5 contradicts 6}

= A is evident to you.

$O\underline{u}$:A = It's obligatory (rationally required) that you believe A.

= Insofar as intellectual considerations are concerned (including your experiences), you ought to believe A.

= A is reasonable for you to believe.

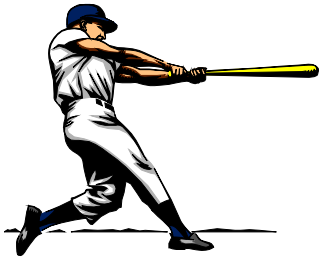
$R\underline{u}$:A = It's all right (rationally permissible) that you believe A.

= Insofar as intellectual considerations are concerned (including your experiences), it would be all right for you to believe A.

It would be unreasonable for you to believe A	=	~R$\underline{u}$:A
It's obligatory that you not believe A	=	O~$\underline{u}$:A
It would be reasonable for you to take no position on A	=	R(~$\underline{u}$:A • ~$\underline{u}$:~A)
It's evident to you that if A then B	=	O$\underline{u}$:(A ⊃ B)
If it's evident to you that A, then it's evident to you that B	=	(O$\underline{u}$:A ⊃ O$\underline{u}$:B)
You ought not to combine believing A with believing not-A	=	O~($\underline{u}$:A • $\underline{u}$:~A)

knowledge	=	*evident true belief* [roughly]
You know that A	=	A is evident to you, A is true, & you believe A.
uKA	=	(O$\underline{u}$:A • (A • u:A))

Hub	=	You hit the ball.
Hub	=	Hit the ball.
OHub	=	You ought to hit the ball.
RHub	=	It's all right for you to hit the ball.



u:Hub	=	You believe that you'll hit the ball.
u:Hub	=	Believe that you'll hit the ball.
u:Hub	=	You act (with the intention) to hit the ball.
u:Hub	=	Act (with the intention) to hit the ball.

Ou:Hub	=	You ought to believe (insofar as your evidence goes) that you'll hit the ball = It's evident to you that you'll hit the ball.
Ru:Hub	=	It's all right (reasonable) for you to believe that you'll hit the ball (insofar as your evidence goes).

Valid

Theism is evident for you.
∴ Atheism is unreasonable for you.

	1		O<u>u</u>:G
	[	∴	~R<u>u</u>:~G
*	2	┌	asm: R<u>u</u>:~G
	3	│	D ∴ <u>u</u>:~G {from 2}
	4	│	D ∴ <u>u</u>:G {from 1}
	5	│	**Du ∴ ~G {from 3} ⇦ B+**
	6	└	**Du ∴ G {from 4} ⇦ B+**
	7	∴	~R<u>u</u>:~G {from 2; 5 contradicts 6}

1. Reverse squiggles (quantificational/modal/deontic).
2. Drop weak operators, using new things: ~<u>u</u>: R (∃x) ◇
3. Lastly, drop strong operators, using old things (if you have them): <u>u</u>: O (x) □

[∴ O~(u:O~Au • u:Au) Valid

* 1 ┌ asm: ~O~(u:O~Au • u:Au)
* 2 │ ∴ R(u:O~Au • u:Au) {from 1}
* 3 │ D ∴ (u:O~Au • u:Au) {from 2}
4 │ D ∴ u:O~Au {from 3}
5 │ D ∴ u:Au {from 3}
6 │ Du ∴ O~Au {from 4} ⇦ **B+**
7 │ **Du ∴ Au {from 5}** ⇦ **B+**
8 └ **Du ∴ ~Au {from 6}**
9 ∴ O~(u:O~Au • u:Au) {from 1; 7 contra 8}

You ought not to combine *believing* that it's wrong for you to do A with *acting* to do A.

1. Reverse squiggles (quantificational/modal/deontic).
2. Drop weak operators, using new things: ~u: R (∃x) ◇
3. Lastly, drop strong operators, using old things (if you have them): u: O (x) □

Our belief logic is oversimplified in three ways. A more sophisticated belief logic would:

- add qualifications to the implicit "One ought to be consistent" axiom and the derived consistency norms,
- perhaps qualify the conjunctivity principle (because of the lottery paradox), and
- add a second deontic operator O* (for what one ought to believe insofar as intellectual considerations go) distinct from O (for what we ought to do all-things-considered).