| | | |
|---|---|---|
| $\sim P$ | = | Not-P |
| $(P \cdot Q)$ | = | Both P and Q |
| $(P \vee Q)$ | = | Either P or Q |
| $(P \supset Q)$ | = | If P then Q |
| $(P \equiv Q)$ | = | P if and only if Q |

Propositional logic

1. Any capital letter is a wff.
2. The result of prefixing any wff with "$\sim$" is a wff.
3. The result of joining any two wffs by "$\cdot$" or "$\vee$" or "$\supset$" or "$\equiv$" and enclosing the result in parentheses is a wff.

# Two translation rules

Put "(" wherever you see "both," "either," or "if."

Either not A or B $=$ $(\sim A \vee B)$

Not either A or B $=$ $\sim(A \vee B)$

Group together parts on either side of a comma.

If A, then B and C $=$ $(A \supset (B \cdot C))$

If A then B, and C $=$ $((A \supset B) \cdot C)$

| | | |
|---|---|---|
| $\sim P$ | = | Not-P |
| $(P \cdot Q)$ | = | Both P and Q |
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| $(P \equiv Q)$ | = | P if and only if Q |

Propositional translations

Put "(" wherever you see "both," "either," or "if."

Group together parts on either side of a comma.

"I went to Paris *and* I went to Quebec."

| P | Q | $(P \cdot Q)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$(0 \cdot 0) = 0$

$(0 \cdot 1) = 0$

$(1 \cdot 0) = 0$

$(1 \cdot 1) = 1$

"$(P \cdot Q)$" claims that *both* parts are true.

"$(P \cdot Q)$" is a *conjunction*; P and Q are its *conjuncts*.

"I went to Paris *or* I went to Quebec."

| P | Q | $(P \vee Q)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

$(0 \vee 0) = 0$
$(0 \vee 1) = 1$
$(1 \vee 0) = 1$
$(1 \vee 1) = 1$

"$(P \vee Q)$" claims that *at least one* part is true.
"$(P \vee Q)$" is a *disjunction*; P and Q are its *disjuncts*.

# “*If* I went to Paris, *then* I went to Quebec.”

| P | Q | $(P \supset Q)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$(0 \supset 0) = 1$
$(0 \supset 1) = 1$
$(1 \supset 0) = 0$
$(1 \supset 1) = 1$

“$(P \supset Q)$” says we *don’t* have the first part true and the second false.
“$(P \supset Q)$” is a *conditional*; P is the *antecedent* and Q the *consequent*.

| | |
|---|---|
| Falsity implies anything. | $(0 \supset \ \ ) = 1$ |
| Anything implies truth. | $(\ \ \supset 1) = 1$ |
| Truth doesn’t imply falsity. | $(1 \supset 0) = 0$ |

| P | Q | $(P \equiv Q)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$(0 \equiv 0) = 1$
$(0 \equiv 1) = 0$
$(1 \equiv 0) = 0$
$(1 \equiv 1) = 1$

"I went to Paris, *if and only if* I went to Quebec."

"$(P \equiv Q)$" claims that both parts have the *same* truth value.
"$(P \equiv Q)$" is a *biconditional*.

| P | ~P |
|---|---|
| 0 | 1 |
| 1 | 0 |

$\sim 0 = 1$
$\sim 1 = 0$

"I *didn't* go to Paris."

"~P" has the *opposite* value of "P."
"~P" is a *negation*.

# Basic Truth Equivalences

| AND | OR | IF-THEN | IFF | NOT |
|---|---|---|---|---|
| $(0 \cdot 0) = 0$ | $(0 \vee 0) = 0$ | $(0 \supset 0) = 1$ | $(0 \equiv 0) = 1$ | |
| $(0 \cdot 1) = 0$ | $(0 \vee 1) = 1$ | $(0 \supset 1) = 1$ | $(0 \equiv 1) = 0$ | $\sim 0 = 1$ |
| $(1 \cdot 0) = 0$ | $(1 \vee 0) = 1$ | $(1 \supset 0) = 0$ | $(1 \equiv 0) = 0$ | $\sim 1 = 0$ |
| $(1 \cdot 1) = 1$ | $(1 \vee 1) = 1$ | $(1 \supset 1) = 1$ | $(1 \equiv 1) = 1$ | |
| *both* parts are true | *at least one* part is true | we don't have *first true & second false* | both parts have *same* truth value | *reverse* the truth value |

Falsity implies anything.
Anything implies truth.
Truth doesn't imply falsity.

# Truth Evaluations

Assume that A=1 and X=0.

What is the truth value of "∼(A · X)"?

- Plug in "1" and "0" for the letters.
- Simplify from the inside out, until you get "1" or "0."

$$\sim(A \cdot X)$$
$$\sim(1 \cdot 0)$$
$$\sim 0$$
$$1$$

Visualize "∼(1 · 0)" as "∼**(1 · 0)**."

# Unknown Evaluations

Assume that T=1 and F=0 and U=? (unknown).

What is the truth value of "(U • F)"?

"(? • 0)" has to be 0 because:

An AND with one part 0 is 0.

It comes out 0 both ways: (1 • 0) = 0 and (0 • 0) = 0.

# Complex Truth Tables: do one for "$(P \equiv \sim Q)$"

| P | Q | $(P \equiv \sim Q)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$(0 \equiv \sim 0) = (0 \equiv 1) = 0$

$(0 \equiv \sim 1) = (0 \equiv 0) = 1$

$(1 \equiv \sim 0) = (1 \equiv 1) = 1$

$(1 \equiv \sim 1) = (1 \equiv 0) = 0$

- Write the formula: "$(P \equiv \sim Q)$"
- On the left, write each letter in the formula: "P" and "Q"
- Below this, write each truth combination; n letters give $2^n$ truth combinations.
- Figure out the value of the formula on each combination.

| | |
|---|---|
| VALID = No possible case has premises all true and conclusion false. | 1<br>1<br>∴ 0 |

| | |
|---|---|
| 1<br>1<br>∴ 0 | INVALID = Some possible case has premises all true and conclusion false. |

Truth-table test: Construct a truth table showing the truth value of the premises and conclusion for all possible cases. The argument is **VALID** if and only if no possible case has premises all true and conclusion false.

| C D | $(C \supset D)$, | D | ∴ | C | | |
|---|---|---|---|---|---|---|
| 0 0 | 1 | 0 | | 0 | | |
| 0 1 | 1 | 1 | | 0 | ← | **Invalid** |
| 1 0 | 0 | 0 | | 1 | | |
| 1 1 | 1 | 1 | | 1 | | |

On the short-cut truth-table test, evaluate easier wffs first and cross out rows that couldn't have true premises and a false conclusion.

| C D | $(C \supset D)$, | D | ∴ | C |
|---|---|---|---|---|
| ~~0 0~~ | | ~~0~~ | | |
| 0 1 | | 1 | | |
| ~~1 0~~ | | ~~0~~ | | |
| 1 1 | | 1 | | |

First do "D" – and cross out rows that couldn't be 110.

| C | D | (C ⊃ D), | D | ∴ | C |
|---|---|---|---|---|---|
| 0 | 0 | | ~~0~~ | | |
| 0 | 1 | | 1 | | 0 |
| 1 | 0 | | ~~0~~ | | |
| 1 | 1 | | ~~1~~ | | ~~1~~ |

Then do "C" – and cross out rows that couldn't be 110.

| C | D | (C ⊃ D), | D | ∴ | C |
|---|---|---|---|---|---|
| 0 | 0 | | ~~0~~ | | |
| 0 | 1 | 1 | 1 | | 0 |
| 1 | 0 | | ~~0~~ | | |
| 1 | 1 | | ~~1~~ | | ~~1~~ |

Finally, do "(C ⊃ D)."

← **Invalid**

| | |
|---|---|
| VALID = No possible case has premises all true and conclusion false. | 1<br>1<br>$\therefore$ 0 |

| | |
|---|---|
| 1<br>1<br>$\therefore$ 0 | INVALID = Some possible case has premises all true and conclusion false. |

Truth-assignment test: Set each premise to 1 and the conclusion to 0. Figure out the truth value of as many letters as possible. The argument is **VALID** if and only if no possible way to assign 1 and 0 to the letters will keep the premises all 1 and conclusion 0.

$$\begin{array}{lllll}
(L \vee R) = 1 & & (L^0 \vee R^0) = 1 & & \overset{0}{\overline{(L^0 \vee R^0)}} \neq 1 \quad \text{Valid} \\
{\sim}L = 1 & \rightarrow & {\sim}L^0 = 1 & \rightarrow & {\sim}L^0 = 1 \\
\therefore\ R = 0 & & \therefore\ R^0 = 0 & & \therefore\ R^0 = 0
\end{array}$$

# Harder Translations

| | |
|---|---|
| Translate "but" ("yet," "however," "although," and so on) as "and." | Northwestern played, *but* it won = $(N \cdot W)$ |
| Translate "unless" as "or." | You'll die *unless* you give me your money = $(D \vee M)$ |
| Translate "just if" and "iff" (a logician word) as "if and only if." | I'll take the job *just if* you pay me a million = $(J \equiv M)$ |

The part after "if" ("provided that," "assuming that," and so on) is the if-part (the antecedent, the part before the horseshoe).

Gensler is happy *if* Michigan wins.
= *If* Michigan wins, Gensler is happy.
= $(M \supset H)$

The part after "only if" is the then-part (the consequent, the part after the horseshoe). (Or write "⊃" for "only if.")

You're alive *only if* you have oxygen.
= *Only if* you have oxygen are you alive.
= $(A \supset O) = (\sim O \supset \sim A)$

| | | |
|---|---|---|
| Oxygen is *sufficient for* life | = | $(O \supset L)$ |
| Oxygen is *necessary for* life | = | $(\sim O \supset \sim L)$ |
| Oxygen is *necessary and sufficient for* life | = | $(O \equiv L)$ |

# Harder Translations

- BUT = YET = HOWEVER = ALTHOUGH = AND.
- UNLESS = OR.
- JUST IF = IFF = IF AND ONLY IF.

- The part after "if" ("provided that," "assuming that," and so on) goes *before* the horseshoe.
- The part after "only if" goes *after* the horseshoe.

- Oxygen is *sufficient for* life = $(O \supset L)$
- Oxygen is *necessary for* life = $(\sim O \supset \sim L)$
- Oxygen is *necessary and sufficient for* life = $(O \equiv L)$

# Idiomatic arguments

1. Identify the conclusion.

| *These often indicate premises:* | | *These often indicate conclusions:* |
|---|---|---|
| Because, for, since, after all …<br>I assume that, as we know …<br>For these reasons … | ∴ | Hence, thus, so, therefore …<br>It must be, it can't be …<br>This proves (or shows) that … |

2. Translate into logic, using wffs. Add implicit premises if needed.

3. Test for validity.

# S-Rules

| | | |
|---|---|---|
| AND | $\dfrac{(P \cdot Q)}{P,\ Q}$ | AND statement, so both parts are true. |
| NOR | $\dfrac{\sim(P \vee Q)}{\sim P,\ \sim Q}$ | NOT-EITHER is true, so both parts are false. |
| NIF | $\dfrac{\sim(P \supset Q)}{P,\ \sim Q}$ | FALSE IF-THEN, so first part true, second part false. |

We can simplify AND, NOR, and NIF.

# I-Rules

| | | |
|---|---|---|
| NOT-BOTH | $\dfrac{\sim(P \cdot Q) \quad P}{\sim Q}$ $\quad$ $\dfrac{\sim(P \cdot Q) \quad Q}{\sim P}$ <br> affirm one part | NOT-BOTH are true, this one is, so the other one isn't. |
| OR | $\dfrac{(P \vee Q) \quad \sim P}{Q}$ $\quad$ $\dfrac{(P \vee Q) \quad \sim Q}{P}$ <br> deny one part | At least one is true, this one isn't, so the other one is. |
| IF-THEN | $\dfrac{(P \supset Q) \quad P}{Q}$ $\quad$ $\dfrac{(P \supset Q) \quad \sim Q}{\sim P}$ <br> affirm $1^{st}$ or deny $2^{nd}$ | IF-THEN, affirm the first, so affirm the second. <br> IF-THEN, deny the second, so deny the first. |

# S- and I-Rules

**AND**

$$\frac{(P \cdot Q)}{P, Q}$$

**NOR**

$$\frac{\sim(P \vee Q)}{\sim P, \sim Q}$$

**NIF**

$$\frac{\sim(P \supset Q)}{P, \sim Q}$$

**NOT-BOTH**

$$\frac{\begin{array}{c}\sim(P \cdot Q)\\ P\end{array}}{\sim Q} \qquad \frac{\begin{array}{c}\sim(P \cdot Q)\\ Q\end{array}}{\sim P}$$

affirm one part

**OR**

$$\frac{\begin{array}{c}(P \vee Q)\\ \sim P\end{array}}{Q} \qquad \frac{\begin{array}{c}(P \vee Q)\\ \sim Q\end{array}}{P}$$

deny one part

**IF-THEN**

$$\frac{\begin{array}{c}(P \supset Q)\\ P\end{array}}{Q} \qquad \frac{\begin{array}{c}(P \supset Q)\\ \sim Q\end{array}}{\sim P}$$

affirm $1^{st}$ or deny $2^{nd}$

# Extended S- and I-rule inferences

$$\frac{\sim((A \cdot B) \supset \sim C)}{(A \cdot B),\ C}$$

FALSE IF-THEN,
so first part true,
second part false.

$$\frac{\sim((A \cdot B) \supset \sim C)}{(A \cdot B),\ C}$$

$$\frac{\begin{array}{c}((A \cdot B) \supset \sim C)\\ \sim(A \cdot B)\end{array}}{\text{nil}}$$

IF-THEN:
need first part true
or second part false

$$\frac{\begin{array}{c}((A \cdot B) \supset \sim C)\\ \sim(A \cdot B)\end{array}}{\text{nil}}$$

# Logic gates and computers

A ➔
B ➔ AND-GATE ➔ (A • B)

| | | |
|---|---|---|
| A | B | (A • B) |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

An AND-GATE gives an output voltage if and only if both inputs have an input voltage.