Quantificational Logic

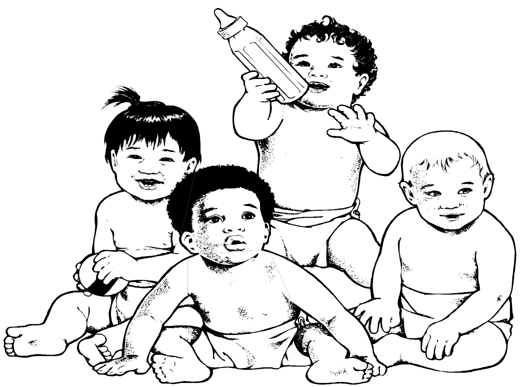
Ir	=	Romeo is Italian.
Ix	=	x is Italian.
(x)Ix	=	For all x, x is Italian.
	=	All are Italian.
(∃x)Ix	=	For some x, x is Italian.
	=	Some are Italian.

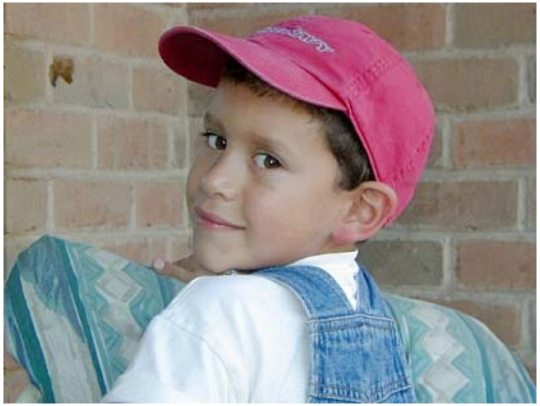
Use capital letters for *general terms* (terms that *describe* or put in a *category*):

B = a cute baby
C = charming
R = rides a bicycle

Use small letters for *singular terms* (terms that pick out a *specific* person or thing):

b = the world's cutest baby
c = this child
w = William Gensler





A capital letter alone (not followed by small letters) represents a *statement*.	S = *It is snowing*.

A capital letter followed by a single small letter represents a *general term*.	Ir = Romeo is *Italian*.

A small letter from "a" to "w" is a *constant* – and stands for a specific person or thing.	Ir = *Romeo* is Italian.

A small letter from "x" to "z" is a *variable* – and doesn't stand for a specific person or thing.	Ix = *x* is Italian.

"(x)" is a *universal quantifier*. It claims that the following formula is true for *all* values of x.

(x)Ix = For all x, x is Italian.
= All are Italian.

"(∃x)" is an *existential quantifier*. It claims that the following formula is true for *at least one* value of x.

(∃x)Ix = For some x, x is Italian.
= Some are Italian.

1. The result of writing a capital letter and then a small letter is a wff.
2. The result of writing a quantifier and then a wff is a wff.

If the English begins with	then begin the wff with
all (every)	(x)
not all (not every)	$\sim(x)$
some	$(\exists x)$
no	$\sim(\exists x)$

All are Italian $=$ $(x)Ix$

Not all are Italian $=$ $\sim(x)Ix$

Some are Italian $=$ $(\exists x)Ix$

No one is Italian $=$ $\sim(\exists x)Ix$

All are rich or Italian $=$ $(x)(Rx \vee Ix)$

Not everyone is non-Italian $=$ $\sim(x)\sim Ix$

Some aren't rich $=$ $(\exists x)\sim Rx$

No one is rich and non-Italian $=$ $\sim(\exists x)(Rx \cdot \sim Ix)$

If the sentence doesn't specify the connective:

with "all … is …," use "$\supset$" for the *middle* connective.	*otherwise* use "•" for the connective.

All Italians are lovers
= $(x)(Ix \supset Lx)$
= For all x, *if* x is Italian *then* x is a lover.

Some Italians are lovers
= $(\exists x)(Ix \cdot Lx)$
= For some x, x is Italian *and* x is a lover.

No Italians are lovers
= $\sim(\exists x)(Ix \cdot Lx)$
= It is not the case that, for some x, x is Italian *and* x is a lover.

All rich Italians are lovers
= $(x)((Rx \cdot Ix) \supset Lx)$
= For all x, *if* x is rich *and* Italian, *then* x is a lover.

Quantificational Logic

Ir	=	Romeo is Italian.		
Ix	=	x is Italian.		
(x)Ix	=	All are Italian	=	For all x, x is Italian.
(∃x)Ix	=	Some are Italian	=	For some x, x is Italian.

If the English begins with:	➔	all (every)	not all	some	no
then begin the wff with:	➔	(x)	~(x)	(∃x)	~(∃x)

With "all … is …," use "⊃" for the *middle* connective.

Otherwise use "•" for the connective.

Quantificational Inference Rules

First reverse squiggles	$\sim(x)Fx \rightarrow (\exists x)\sim Fx$ $\sim(\exists x)Fx \rightarrow (x)\sim Fx$	*
and drop existentials.	$(\exists x)Fx \rightarrow Fa$, use a *new* constant	*
Lastly, drop universals.	$(x)Fx \rightarrow Fa$, use any constant	Don't star

	1	(x)(Fx · Gx)	**Valid**
		[∴ (x)Fx	
*	2	┌ asm: ~(x)Fx	
*	3	│ ∴ (∃x)~Fx {from 2}	🡐 reverse squiggles
	4	│ ∴ ~Fa {from 3}	🡐 drop existentials
	5	│ ∴ (Fa · Ga) {from 1}	🡐 drop universals
	6	└ ∴ Fa {from 5}	
	7	∴ (x)Fx {from 2; 4 contradicts 6}	

1. Reverse squiggles.
2. Drop initial existentials, using a new letter each time.
3. Lastly, drop each initial universal once for each old letter. (Only use a new letter if you've done everything else possible, including further assumptions if needed, and still have no old letters.)

	1	$(x)(Lx \supset Fx)$		
*	2	$(\exists x)Lx$		
	[$\therefore$	$(x)Fx$		
*	3	asm: $\sim(x)Fx$		
*	4	$\therefore (\exists x)\sim Fx$	{from 3}	
	5	$\therefore La$	{from 2}	
	6	$\therefore \sim Fb$	{from 4}	
*	7	$\therefore (La \supset Fa)$	{from 1}	
*	8	$\therefore (Lb \supset Fb)$	{from 1}	
	9	$\therefore Fa$	{from 5 and 7}	
	10	$\therefore \sim Lb$	{from 6 and 8}	

Invalid

a, b

$$\boxed{\begin{array}{c} La,\ Fa \\ \sim Lb,\ \sim Fb \end{array}}$$

Reverse squiggles, drop existentials, drop universals.
If you can't get a contradiction, construct a refutation.

1	$(x)(Lx \supset Fx) = 1$	
2	$(\exists x)Lx = 1$	
	$[\therefore (x)Fx = 0$	

Invalid

a, b

La, Fa
~Lb, ~Fb

An *existential* wff is true if and only if *at least one case* is true.	A *universal* wff is true if and only if *all cases* are true.

If a wff doesn't start with a quantifier: evaluate each subformula that starts with a quantifier, and then substitute "1" or "0" for it:

$\sim(x)Fx$	$\sim(x)(Lx \supset Fx)$	$((\exists x)Fx \supset (x)Lx)$
$\sim\mathbf{(x)Fx}$	$\sim\mathbf{(x)(Lx \supset Fx)}$	$(\mathbf{(\exists x)Fx} \supset \mathbf{(x)Lx})$
$= \sim 0$	$= \sim 1$	$= (1 \supset 0)$
$= 1$	$= 0$	$= 0$

Valid

	1	(x)(Fx · Gx)	
		[∴ (x)Fx	
*	2	┌ asm: ~(x)Fx	
*	3	│ ∴ (∃x)~Fx	{from 2}
	4	│ ∴ ~Fa	{from 3}
	5	│ ∴ (Fa · Ga)	{from 1}
	6	└ ∴ Fa	{from 5}
	7	∴ (x)Fx	{from 2; 4 contradicts 6}

Invalid

a, b

	1	(x)(Lx ⊃ Fx)	
*	2	(∃x)Lx	
		[∴ (x)Fx	
*	3	asm: ~(x)Fx	
*	4	∴ (∃x)~Fx	{from 3}
	5	∴ La	{from 2}
	6	∴ ~Fb	{from 4}
*	7	∴ (La ⊃ Fa)	{from 1}
*	8	∴ (Lb ⊃ Fb)	{from 1}
	9	∴ Fa	{from 5 and 7}
	10	∴ ~Lb	{from 6 and 8}

La, Fa
~Lb, ~Fb

1. Reverse squiggles.
2. Drop initial existentials, using a new letter each time.
3. Lastly, drop each initial universal once for each old letter. (Only use a new letter if you've done everything else possible, including further assumptions if needed, and still have no old letters.)
4. If you can't get a contradiction, construct a refutation.

Harder translations add statement letters, individual constants, and non-initial or multiple quantifiers:

$(S \supset Cr)$	=	If it's snowing, then Romeo is cold.
$((x)Ix \supset (x)Lx)$	=	If all are Italian, then all are lovers.

Use a separate quantifier for each "all," "some," and "no"; and place the quantifiers to mirror where they occur in English:

Wherever the English has	put this in the wff
all (every)	(x)
not all (not every)	$\sim(x)$
some	$(\exists x)$
no	$\sim(\exists x)$

Translate this right now:

"If all Greeks are mortal and Socrates is Greek, then someone is mortal and it will rain."

$$(((x)(Gx \supset Mx) \cdot Gs) \supset ((\exists x)Mx \cdot R))$$

"If all Greeks are mortal and Socrates is Greek, then someone is mortal and it will rain."

To translate “any”:

First rephrase the sentence so it means the same thing but doesn’t use “any”; then translate the second sentence.	or	Put a “(x)” at the *beginning* of the wff, regardless of where “any” occurs in the sentence.

Not anyone is rich = $\sim(\exists x)Rx$ = $(x)\sim Rx$

Not any Italian is a lover = $\sim(\exists x)(Ix \cdot Lx)$ = $(x)\sim(Ix \cdot Lx)$

If anyone is just, there will be peace = $((\exists x)Jx \supset P)$ = $(x)(Jx \supset P)$

Harder translations add statement letters, individual constants, and non-initial or multiple quantifiers:

$(S \supset Cr)$

$((x)Ix \supset (x)Lx)$

Wherever the English has:	➔	all (every)	not all	some	no
put this in the formula:	➔	(x)	$\sim(x)$	$(\exists x)$	$\sim(\exists x)$

With "all … is …," use "⊃" for the *middle* connective.

Otherwise use "•" for the connective.

To translate a sentence with "any":

- Rephrase the sentence so it means the same thing but doesn't use "any"; then translate the second sentence.
- OR: Put a "(x)" at the *beginning* of the wff, regardless of where "any" occurs in the sentence.

Proofs with harder formulas:

- use statement letters, individual constants, or non-initial or multiple quantifiers,
- often require multiple assumptions, but
- require no new inference rules.

Remember to drop only initial quantifiers.

"$((x)Fx \supset (x)Gx)$" is an if-then and follows the if-then rules.

The Two Great Commandments:

(1) Thou shalt drop only initial quantifiers.

(2) Thou shalt use a new letter when dropping $(\exists x)$.

Do these problems now:

$\sim(x)(Fx \vee Sx)$
$\therefore ((\exists x)\sim Sx \cdot \sim Fa)$

$(\sim(\exists x)Gx \supset (\exists x)Fx)$
$\therefore (\exists x)(\sim Fx \supset Gx)$

1. Reverse squiggles.
2. Drop initial existentials, using a new letter each time.
3. Lastly, drop each initial universal once for each old letter. (Only use a new letter if you've done everything else possible, including further assumptions if needed, and still have no old letters.)
4. If you can't get a contradiction, construct a refutation.

Do this problem now:

$$\sim(x)(Fx \vee Sx)$$
$$\therefore ((\exists x)\sim Sx \cdot \sim Fa)$$

1. Reverse squiggles.
2. Drop initial existentials, using a new letter each time.
3. Lastly, drop each initial universal once for each old letter. (Only use a new letter if you've done everything else possible, including further assumptions if needed, and still have no old letters.)
4. If you can't get a contradiction, construct a refutation.